01 A. Sets Reading, 2.1 Sets

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Cornell Notes	**Topic:** 01 A. Sets-Reading 2.1 Sets	Course: BSc Computer Science Class: Discrete Mathematics-Reading Date: October 9, 2019

Essential Question:

What are sets (how can we write them and "measure" them) and what are "elements" within a set, to that what are sets within sets, the powerset of a set and the Cartesian Product of a set?

Questions/Cues:

- What is the fundamental discrete structure on which all other discrete structures are built?
- What are sets used for?
- What is the intuitive definition of a set?
- What are ways to describe a set?
- What are some sets that play an important role in discrete mathematics?
- What are the intervals for real numbers?
- Can sets have other sets as elements and/or members?
- What does it mean when two sets are equal?
- What is a set with no elements called?
- Can sets be represented graphically?
- What is a subset of a set?
- What is a proper subset of a set?
- What is the size and/or cardinality of a set?
- Is a set infinite?
- What is a powerset of a set?
- What are ordered n-tuples?
- What is a Cartesian Product of a set?
- What is the Cartesian product more than two sets?
- What is the notation to denote the Cartesian Product of a set with itself?

Notes

- The Set = Most Fundamental discrete structure, foundation for other discrete structures
- Sets used to group objs' together
    - Often but not always, objs' in set have similar properties

- Nothing prevents set from having seemingly unrelated elements { a, 2, Fred, New Jersey }
    - Order doesn't matter and duplicates don't either (just count them once)
- Set = unordered collection of objs', called *elements* or *members* of set
    - said to *contain* its elements
    - $a \in A$ to denote a is element of set A
    - $a \notin A$ to denote a is not element of set A
    - Uppercase letter to show set and lowercase letter to showcase elements of sets
- List all members of set, members listed between braces a.k.a. **roster method** or **listing method**
    - X = { a, b, c, d }
    - used to describe set without all members, with some members listed then ellipses(...) when pattern is obvious
- Second is **set builder notation**, characterize elements in set by property or properties they must have to be members
    - $O = \{x \mid x$ is an odd positive integer less than $10\}$
    - used to describe sets when impossible to list all elements in set
    - $\mathbf{Q}_+ = \{x \in \mathbf{R} \mid x = p/q$, for some positive integers p and $q\}$
- $\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of **natural numbers**

  $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of **integers**

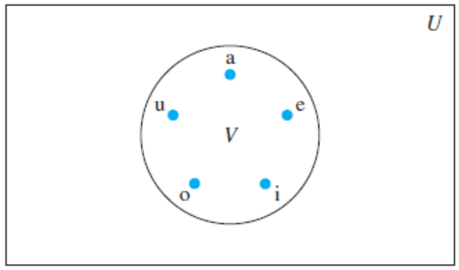
  $\mathbf{Z}_+ = \{1, 2, 3, \ldots\}$, the set of **positive integers**

  $\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}$, and $q \neq 0\}$, the set of **rational numbers**

  R, the set of **real numbers**
  R+, the set of **positive real numbers**
  C, the set of **complex numbers**

- $a < b$,
    - $[a, b] = \{x \mid a \leq x \leq b\}$
    - $[a, b) = \{x \mid a \leq x < b\}$
    - $(a, b] = \{x \mid a < x \leq b\}$
    - $(a, b) = \{x \mid a < x < b\}$
        - [a, b] = closed interval from a to b
        - (a, b) = open interval from a to b
- Sets can have other sets as members ie. set {**N**,**Z**,**Q**,**R**}
- Sets are equal ↔ have same elements, A and B are sets, then A and B are equal **if and only if**
    - $A \subseteq B$ *and* $B \subseteq A$
    - $\forall x (x \in A \leftrightarrow x \in B)$
    - ∀ x (x ∈ A ↔ x ∈ B)
        - ∀ = for all instances of [variable]
        - ∈ = element of
        - ↔ = if and only if
- ∅ = empty or null set, no elements, empty also denoted { } (pairs of braces, empty set encloses all element in this set)
    - set of elements with certain properties = null set, ie. set of all positive ints greater than their squares
- One element set = singleton set
    - $\emptyset \neq \{ \emptyset \}$, because single element of set { ∅ } is empty set itself!
    - ie. ∅ = empty folder, { ∅ } = folder with one folder inside which is empty folder.
- Sets (graphically) called Venn diagrams
    - Universal set **U** contains all objs' under consideration, rep'ed by rectangle

- circles or other geo shapes used to rep sets
- Sometimes points used to rep elements
- Venn diagrams used to indicate relationships between sets.



- Set A is subset of B ↔ every element of A is also element of B.
  - $A \subseteq B$, notation that A is subset of set B
  - $\forall x\ (x \in A \rightarrow x \in B)$
    - To show subset, if x belongs to A then x belongs to B
    - To show not subset, find $x \in A$ such that $x \notin B$
  - Every set S, guaranteed at least 2 subsets, empty set and set S itself; $\emptyset \subseteq S$ and $S \subseteq S$
- Proper subset = Set A is subset of set B, $A \neq B$; $A \subset B$
  - $A \subseteq B$
  - must exist element x of B that not element of A
    - $\forall x\ (x \in A \rightarrow x \in B) \wedge \exists x\ (x \in B \wedge x \notin A)$
      - $\rightarrow$ = if...then
      - $\wedge$ = And
      - $\exists$ = there exists
- Cardinality (Size of Set) for S = $|S|$, S is **finite** set
  - exactly *n* distinct elements, n is non-negative
  - n in this case is cardinality of S
- A set is said to be infinite if it is not finite.
- Given set S, powerset of S = set of all subsets of S
  - denoted by $P(S)$
  - $P(\emptyset) = \{\emptyset\}$
  - $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$
  - If set has *n* elements, then power set has 2^n elements

- ordered n-tuple $(a_1, a_2, \ldots, a_n)$= ordered collection, a_1 as 1st element, a_2 as 2nd element,.... and a_n as *n*th element
  - to represent order collections
    - two ordered n-tuples equal↔ $(a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n)$ ^ ↔ $a_i = b_i$, for $i = 1, 2, \ldots, n$
    - ordered 2-tuples = **ordered pairs**
      - ordered pairs (a, b) and (c, d) equal ↔ a = c and b = d
        - ****Note** (a, b) $\neq$ (b, a), unless a = b
- A and B be sets, Cartesian product of A and B, $A \times B$, is set of all ordered pairs (a, b); $a \in A$ and $b \in B$
  - $A \times B = \{ (a, b) \mid a \in A \wedge b \in B \}$
    - ie. Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?
      - Ans: $A \times B = \{ (1, a), (1, b), (1, c), (2, a), (2, b), (2, c) \}$

- **Note** $A \times B \neq B \times A$, unless A = ∅ or B = ∅
  so $A \times B = \emptyset$ or A = B

- Cartesian product of sets $A_1, A_2, \ldots, A_n$, denoted $A_1 \times A_2 \times \cdots \times A_n$, is set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$
  - a_i belongs to A_i for i = 1, 2, . . . , n
  - $A_1 \times A_2 \times \cdots \times A_n = \{ (a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n \}$
    - ie. Cartesian product A × B × C, where A = {0, 1}, B = {1, 2}, and C = {0, 1, 2} ?
      - A × B × C = all ordered tuples (a, b, c), a ∈ A, b ∈ B, and c ∈ C

A × B × C = { (0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2) }

    - A, B and C are sets, (A × B) × C is not same as A × B × C

- $A^2 = A \times A$, $A^3 = A \times A \times A$, $A^4 = A \times A \times A \times A$

Summary

In this week, we've defined what sets are. Alongside this, we've looked at what makes up a set, how to count the elements of a set and most importantly how to denote a set in various way such set builder notation. Furthermore, we've looked at how sets can be nested other sets, special sets used in mathematics, power sets or set of all subsets within a set, and the Cartesian Product of a set.